LOGICAL AND CRITICAL ANALYSIS OF THE FOUNDATIONS OF PURE AND APPLIED MATHEMATICS

Swatilekha Nag

Abstract: The analysis of the foundations of pure and applied mathematics is proposed. The unity of formal logic and of rational Dialectics is methodologically basis of the analysis. The main result is as follows: the concept of "mathematical quantity"-Starting and central concept in mathematics – is meaningless, erroneous and inadmissible one because it represents the following formal-logical and dialectical-materialistic errors: negation of existence of measure of material object. The correct metrical- geometrical and physical interpretations of mathematical relationship show that the set of standard mathematical functions represents error, mistake and blunder. Therefore, pure mathematics does not satisfy the general-scientific criterion of truth: practice is criterion of truth. The obtained results lead to the conclusion that the generally accepted foundations of pure mathematics are a false one, and they should be reconsidered. Results of the critical analysis of the standard foundations of mathematics applied to problems in physics are discussed.

Keyword: mathematics, geometry, mathematical physics, physics, engineering, formal logic, dialectics, philosophy of science.

INTRODUCTION

Foundation of mathematics is the study o the philosophical and logical and/or algorithmic basis of mathematics, or, in a broader sense, the mathematical investigation of what underlies the philosophical theories concerning the nature of mathematics. In this latter sense, the distinction between foundations of mathematics and philosophy of mathematics turns out to be quite vague. Foundations of mathematics can be conceived as the study of the basis mathematical concepts (number, geometrical, figure, set, function, etc) and how they form hierarchies of more complex structures and concepts, especially the fundamentally important structures that form the language o mathematics (formulas, theories and their models giving a meaning of formulas, definitions, proof, algorithms etc) also called mathematical concepts, with an philosophy of mathematics; the abstract nature of mathematical objects presents special philosophical challenges.

The close connection between mathematics and philosophy has long be recognized by practitioners of both disciplines. The apparent timelessness of mathematical truth, the exactness and objective nature of its concepts, its applicability to the phenomena of the empirical world- explicating such facts presents philosophy with some of its subtlest problems. Let me begin by reminding you of some celebrated past attempts made by philosophers and mathematicians to explicate the nature of mathematics.

Classical views on the nature of Mathematics

1 Plato included mathematical entities- numbers and the objects of pure geometry such as points, lines, and circles- among the well-defined, independently existing eternal objects he called Forms. It

is the fact that mathematical statements refer to these definite Forms that enables such statements to be true or false.

2 Plato's pupil and philosophical successor Aristotle on the other hand, rejected the notion of Forms being separate from empirical objects, and maintained instead that the Forms constitute parts o objects. Forms are grasped by the mind through a process of abstraction from sensible objects, but they do not thereby attain an autonomous existence detached from these latter.

3 Leibnitz divided all true propositions, including those of mathematics, into two types: truths of fact, and truths of reason, also known as contingent and analytic truths, respectively. According to Leibniz, true mathematical propositions are truth of reason their truth is therefore just logical truth: their denial would be logically impossible. Mathematical propositions do not have a special "mathematical" content-as they did for Plato and Aristotle-and so true mathematical propositions are true in all possible worlds, that is, they are necessarily true.

As is well known, mathematics is widely and successfully used in the natural sciences. However, it does not mean that the problem of validity of pure mathematics is now completely solved, or that the foundations of mathematics are not in need of formal-logical and dialectical-materialistic analysis. Critical analysis within the framework of the correct methodological basis shows [1-12] that the foundations of theoretical physics and of mathematics (for example, classical geometry, the Pythagorean theorem, differential and integral calculus, vector calculus, trigonometry, theory of negative numbers) contain formal-logical errors.

Results of the critical analysis of the standard foundations of mathematics applied to problems in physics are discussed. The unity of formal logic and of rational dialectics is methodological basis of the analysis. The main result is as follows: the concept of "mathematical quantity" – central concept of mathematics – is meaningless, erroneous, and inadmissible one because it represents the following formal-logical and dialectical-materialistic errors: negation of the existence of the essential sign of a concept (i.e., negation the existence of the essence of the concept) and negation of the existence of measure of material object. The obtained results lead to the conclusion that the generally accepted foundations of mathematics should be reconsidered.

1. THE METHODOLOGICAL BASIS

As is known, correct methodological basis of science is the unity of formal logic and of rational dialectics. Use the correct methodological basis is a necessary condition for correct analysis to make distinction between truth and falsehood. However, this fact is ignored by majority of scientists until now. Therefore, the main propositions of formal logic and of rational dialectics which are used in the present work must be stated.

1.1. The basic principles of formal logic

1. Formal logic is science of the laws of correct thinking as well as means of cognition of reality. Correct thinking represents uncontradictory, coherent, consistent, and sequential thinking. The conclusions resulting from correct thinking are true statements which reflect correctly the objective reality in the process of scientific cognition of the world. The basic formal-logical laws are the

following four laws: the law of identity, the law of lack (absence) of contradiction, the law of excluded middle, the law of sufficient reason.

2. Thinking is the highest form of human cognitive activity which represents the process of reflection of objective reality in human consciousness. Human thinking is performed with the help of concepts and has different forms.

3. The form of thought reflecting and fixing the essential sings (features) of things, objects, and phenomena of reality is called concept. In other words, the concept is the thought that reflects things, objects from viewpoint of the general and essential signs (features). (Thing is an object that can be in relation to anything or have some property).

4. The essential sings (features) of the concept are chosen (are singled out) in objects and phenomena by thought. The essential sings (features) characterize the objects of given kind. Non-essential sings (features) do not characterize the objects of given kind. The characteristic which is used to determine similarity or difference of objects of thought is called essential sing (feature). In the most general view, sings (features) of objects can be reduced to properties (for example, large, small, white, black, good, bad, soft, hard, etc.), states (for example, state of rest, state of motion, energetic state, equilibrium state, etc.), actions (for example, it works, he reads, she performs her duties, etc.), and results of actions (for example, have scored success, have benefited, etc.), etc.

5. The first basic form of thought is a concept. Concepts are formed (created) with the help of logical methods such as analysis and synthesis, abstraction and generalization. Analysis is the mental decomposition (dissection) of the object of thought in terms of the elements, the choice (separation) of either sing, and the consideration of it separately. Analysis does not give knowledge of object or of phenomenon as a whole. Synthesis is the mental integration (association, combination, junction) of the elements of the object or of the phenomenon. Synthesis provides knowledge of object or of phenomenon as a whole (as a unity of parts, as a system). But this knowledge is not the reliable and complete one. Abstraction is the mental separation, the mental extraction of the certain, the essential sings (features) of object or of phenomenon and passing over all other sings (i.e., abandonment of all other sings (features) without consideration). Generalization is the mental transition from sings (features) of individual, separate, single objects to sings (features) belonging to whole groups (classes) of these objects. Abstraction is the mental separation, the mental extraction of the certain, the essential sings (features) of object or of phenomenon and passing over all other sings (i.e., abandonment of all other sings (features) without consideration). Generalization is the mental transition from sings (features) of individual, separate, single objects to sings (features) belonging to whole groups (classes) of these objects.

6. All the concepts can be divided into the following separate types: single concepts and general concepts. The concept that relates to the only one certain object, separate phenomenon, separate event is called single (individual) concept. The concept that embraces (covers) a group (class) of similar things, objects is called general concept.

7. Each concept has two aspects: the scope (volume) of the concept and the content of the concept. The scope (volume) of the concept is all the objects and phenomena which can be embraced (covers) by given concept. The scope (volume) of the general concepts is expressed in the form of a logical class. The concept content is a set of all essential features of objects.

8. All the concepts can be divided into the following separate types: concrete concepts and abstract concepts. Concrete concept is the concept that relates to groups, classes of objects, phenomena or to the separate objects, phenomena. Abstract concept is the concept of properties of objects or phenomena if these properties are taken as the separate (independent) object of thought and are abstracted from objects.

9. There is a special kind of concepts that is called categories. Categories are the scientific concepts reflecting the most common properties of objects and phenomena, the most common and essential relations and connections in reality. For example, the concepts of “matter”, “movement”, “content”, “form”, “causality”, "freedom", “necessity”, “essence”, “phenomenon” are the categories.

10. There are the following relations between the concepts: identity relation; relation of subordination; relation of collateral subordination; relation of partial coincidence; relation of disagreement. (For example, the relation of disagreement exists between contradictory concepts and opposite concepts).

11. The second, more complicated, form of thought is a proposition. The proposition is the logical form of expression of thought. The proposition is the logical content of grammatical sentence. The proposition is a statement about the objects and phenomena of objective reality. The statement states the existence or absence of certain features (sings) of objects and of phenomena. The proposition has the following two properties: (a) the proposition either asserts or denies (negates); (b) the proposition is either true or false. The proposition is always assertion or negation. The proposition is true if it reflects correctly the reality; and the proposition is false if it reflects incorrectly the reality. Every proposition represents a system of concepts. There are three elements in every proposition: subject, predicate, connective. The subject of the proposition is that what one states about. The predicate of the proposition is that what one states on the subject. The connective is an indication of the relation between subject and predicate. In any proposition, subject and predicate are concepts connected by connective. The connective in any proposition expressed by the word “is” or “is not”.

12. The third form of thought is an inference. The inference represents connection of propositions, which makes it possible to derive a new proposition from given one or more propositions. Those propositions from which one derives the new proposition are called premises, and the new proposition derived from the premises is called conclusion. Relation between the premises and the conclusion is relation between reason (basis) and consequence (logical corollary): the premises are the reason (basis) from which the conclusion follows as a consequence (logical corollary). Consequently, the inference is based on the law of sufficient reason. Depending on number of premises, all the inferences are divided into two groups: immediate inferences and mediated inferences. The immediate inference is the inference in which the conclusion is consequence of one premise. The mediated inference is the conclusion in which a new proposition is derived from two or more propositions.

13. The mediated inferences can be of two types: deductive and inductive. The mediated deductive inference is called syllogism if a conclusion is derived from two premises. The inference is called inductive inference if the premises indicate features of separate objects or groups of separate objects, and the conclusion is extended to other objects of the same kind. Deduction and induction

are in inseparable connection with each other and supplement each other. Mathematics uses mainly method of deduction.

14. Scientific induction is based on the determination of the causes. Therefore, the problem of causal connection of phenomena is important for scientific induction. The causal connection of phenomena is that one phenomenon is a cause another phenomenon, and a change in the first phenomenon entails a change in the second phenomenon too. The phenomenon which necessarily entails another phenomenon is called cause, and the second phenomenon which is entailed by this cause is called effect of this cause. Thus, the connection of cause and of effect is a connection of two phenomena, two facts. In order to determine the cause of the phenomenon studied, one should use two basic logical methods of the inductive research: inter comparison of the circumstances in which given phenomenon occurs; comparison of these circumstances (in which given phenomenon occurs) with other circumstances (similar in other relations) in which given phenomenon do not occur.

2. ANALYSIS OF THE CONCEPT OF QUANTITY

The starting (prime, initial, basic) and the central concept of pure and applied mathematics (i.e., the mathematical formalism of the natural sciences) is the concept of quantity. The quantity is a characteristic, property, sign (feature). In pure mathematics, the quantity is the quantitative determinacy expressed by unnamed (dimensionless) numbers. In physics and engineering, the quantity is the quantitative determinacy expressed by named (dimensional) numbers and is called physical quantity.

2.2. The mathematical quantity

As is known, pure mathematics does not operate with the physical quantities. Mathematical concepts and formulations of mathematical propositions and of theories are abstracted from material objects and concrete nature (i.e., physical meaning) of quantities. Mathematical concepts take into account only the numerical values of quantities, i.e., the concepts of “length”, “square”, “volume”, “weight”, “temperature”, “speed”, “force”, etc. are mathematically considered as identical concepts. This implies that length, area, volume, mass, temperature, speed, force, etc. are not the essential signs (features) of the mathematical concepts. In other words, the concept of physical quantity does not exist in pure mathematics; and dimensionless (unnamed, abstract) numbers represent the only sign (feature) of mathematical concepts. This implies that pure mathematics operates with abstract quantity in general, i.e., abstract dimensionless quantity which exists only in thoughts. This abstract dimensionless quantity is called mathematical quantity. In my view, the concept of “mathematical quantity” – the starting (prime, initial, basic) and central concept of pure and applied mathematics – is not free from objections.

2.2.1. Formal-logical objection

Formal-logical objection is as follows.

1. Numbers are a result of measurement of the concrete physical quantity which has dimension and characterizes the determinate single material object. One makes measurement with the help of device destined for measurement of given concrete physical quantity. But measurement of given concrete physical quantity with the help of device which is not destined for measurement of this

concrete physical quantity cannot be made (i.e., numbers do not exist in this case). Moreover, the devices which could measure abstract and dimensionless quantities do not exist. This implies that the dimensionless (unnamed, abstract) numbers do not exist in practice: they exist only in thoughts. Therefore, the mathematical quantity represents fictitious, dummy quantity.

2. Within the framework of formal logic, one can give the following genetic definition of the concept of "mathematical quantity" (As is known, the genetic definition of concept shows how given concept arises). The concept of "mathematical quantity" arises as the result of the following mental operations: (a) abstraction of "quantitative determinacy of the physical quantity" from the "physical quantity" under the assumption that "the quantitative determinacy of the physical quantity" is a independent (separate) object of thought; (b) abstraction of "amount (i.e., abstract number)" from the "quantitative determinacy of the physical quantity" under the assumption that "number (i.e., abstract number)" is a independent (separate) object of thought. In this case, unnamed, abstract numbers represent the only sign (feature) of the concept of "mathematical quantity". However, this sign (feature) is not the essential sign (feature) of the material objects. Furthermore, the content of the concept of "mathematical quantity" is equal to zero, and the scope (denotation, volume) of this concept is infinitely large. This implies that the essence of the concept of "mathematical quantity" is that the concept of "mathematical quantity" contradicts to the standard logical definition of concept. Consequently, the mathematical quantity is a fictitious, dummy quantity.

Thus, the concept of mathematical quantity is vacuous, meaningless, fallacious, and inadmissible concept in science because it represents the following formal-logical and dialectical-materialistic errors: negation of the standard proposition of existence of the essential sign (feature) of concept (i.e., negation of existence of the attribute, essence of concept); and negation of existence of measure of a material object.

3. Relation between the concepts of "abstract physical quantity" and of "mathematical quantity" is the relation of disagreement. Really, the relation of disagreement exists between such concepts which have different scopes (volumes) and contents under the stipulation that the signs (features) entering into the content of one concept ("mathematical quantity") deny (negate) the signs (features) entering into the content of another concept ("abstract physical quantity"). Therefore, the concept of mathematical quantity contradicts to the concept of measure and represents a formal- logical error.

Thus, the formal-logical objection is that mathematical quantity represents fictitious, dummy quantity.

On the Formal-logical Analysis of the foundations of Mathematics applied to problems in Physics

1. As is well known, mathematical physics is the fundamental science of "the application of mathematics to problems in physics and the development o mathematical methods suitable for such applications and or the foundation of physical theories". Mathematical physics arose from the needs of science and practice and has a long history of development. The important significance of this field of science is determined by the contribution of prominent scientists of

past and present times. However, it does not mean that the problem of validity of the achievements of mathematical physics is now completely solved, or that the foundations of mathematics and physics are not in need of analysis within the framework of the correct methodological basis: the unity of formal logic and of rational dialectics. Standard mathematical and physical cannot be considered as scientific truth if there is no formal-logical and dialectical substantiation of it in science.

2. The correct methodological basis represents theoretical generalization of practice and, therefore is the criterion of truth of theories. In this connection, the following questions arise: How can we apply formal logical and dialectical laws to the analysis of the foundations of the special sciences? Do mathematical and physical theories obey the formal-logical and dialectical laws? In my opinion, the formal-logical law of identity and the dialectical-materialistic law of measure (i.e., the law of interrelation, of interdependence, of inter-conditionality of qualitative and quantitative aspects (determinacy) of a material object) can be used in mathematics and physics in the following wording: the left-hand side and the right-hand side of the mathematical (i.e., the quantitative) relationship must have identical qualitative determinacy. As first shown in my published works, the use of this statement leads to the following conclusion: the foundations of theoretical physics and of mathematics are not free from objections because the standard theories do not satisfy the criterion of truth.

3. In my opinion, a thorough understanding of the foundations of mathematics is impossible without a critical analysis of the concept of mathematical quantity – central concept of mathematics. A critical analysis of the concept of mathematical quantity leads to the following conclusion:

   The concept of "mathematical quantity" is the result of the following mental operations: (i) abstraction of the "quantitative determinacy of physical quantity" from the "physical quantity" at that the "quantitative determinacy of physical quantity" is an independent object of thought; (ii) abstraction of the "amount (i.e., abstract number)" from the "quantitative determinacy of physical quantity" at that the "amount (i.e., abstract number)" is an independent object of thought. In this case, unnamed, abstract numbers are the only sign of the "mathematical quantity". This sign is not an essential sign of the material objects. Therefore, the content of the concept of "mathematical quantity" is zero, and the volume of this concept is infinitely large.

   Thus, the concept of mathematical quantity is meaningless, erroneous, and inadmissible concept in science because it represents the following formal-logical and dialectical-materialistic error: negation of the existence of the essential sign of the concept (i.e., negation the existence of the essence of the concept) and negation of the existence of measure of material object.

4. Mathematical theories can be applied to problems in physics if and only if the mathematical relationships are interpreted geometrically or physically. The interpretation is that the mathematical quantities in the standard relationship $y=f(x)$ are associated with geometric (metric) or physical quantities x_M and y_M characterizing the material object M. Interpretation operation represents replacement $x \rightarrow x_M$, $y \rightarrow y_M$ in the relationship $y=f(x)$. In fact, the interpretation is expressed by the identities $x=x_M$, $y=y_M$. In this case, the relationship between the physical quantities has the form $y_M = f(x_M)$ and can be tested in practice. This relationship means that the interpretation leads to the restoration of the measure of the material object

(i.e., the restoration of the unity of qualitative and quantitative determinacy of the material object). Pure mathematics (i.e., mathematics without restored measure) is, according to Einstein, useless science. Therefore, the interpretation is a criterion of truth in pure mathematics.

5. In my published works, analysis of the foundations of mathematics applied to problems in physics was proposed. It was first shown within the correct methodological basis that the foundations of differential and integral calculus, the foundations of vector calculus, the Pythagorean theorem, the foundations of trigonometry, and the foundations of the theory of negative numbers are not free from objection because these standard results contain the formal-logical and dialectical-materialistic errors. Removing these errors leads to the abolition of many standard theories. And the abolition of the standard theories turns science into helpless and barren knowledge.

DISCUSSION

1. As is known, formal logic is the general science of the laws of correct thought. The laws of formal logic represent the theoretical generalization and reflection of practice in human consciousness. Consequently, formal logic exists in human consciousness and practice. Practice is criterion of validity (trueness, truth) of formal logic.

2. Dialectical materialism is the general science of the most common (general) kinds of connections and laws of development of nature, of human society, and of thought. The laws of dialectics represent the theoretical generalization and reflection of practice in human consciousness. Consequently, dialectics exists in human consciousness and practice. Practice is criterion of validity (trueness, truth) of dialectics.

3. The only correct methodological basis of science is the unity of formal logic and of rational dialectics. Mathematics is a science if and only if its foundations are formulated within the framework of correct methodological basis.

4. Pure mathematics is partial, special, non-general, non-common, and abstract science. Today, there is no complete understanding of the essence of pure mathematics. In my opinion, the essence of mathematics can be understood only within the framework of correct methodological basis. The metric-geometrical and physical interpretations of mathematical concepts, theorems, and theories within the framework of correct methodological basis disclose the essence of mathematics.

5. The concept of mathematical quantity is starting and central concept in mathematics. As the critical analysis shows, mathematical quantity and relationships between mathematical quantities do not satisfy the criterion of truth. In order that mathematical quantity and relationships between mathematical quantities satisfy the physical criterion of truth, one must interpret physically mathematical quantity and relationships between mathematical quantities. Interpretation is operation of restoration of measure, i.e., restoration of the unity of qualitative and quantitative determinacy of material object. Mathematics without operation of restoration of measure represents a fiction, a useless intellectual game.

6. In my opinion, the correct definition of the essence of mathematics is as follows. Correct mathematics is a concretization of the basic principles of materialistic dialectics: the correct mathematics is the science of quantitative determinacy of the measure of a material object.

CONCLUSION

Thus, the correct analysis of the foundations of mathematics is possible only within the framework of correct methodological basis: the unity of formal logic and of rational dialectics. The analysis leads to the following results:

1) The concept of mathematical quantity – the starting and the central concept in pure mathematics – is meaningless, mistaken, and inadmissible concept in science because it contradicts to the basic propositions (statements) of formal logic and of materialistic dialectics. The concept of mathematical quantity represents the following the formal-logical and dialectical materialistic errors: negation of the existence of the essential sign (feature) of concept (i.e., negation of existence of the essence (content, intension) of concept) and negation of existence of measure of material object.

2) Criterion of validity (trueness, truth) of pure mathematics is metric-geometrical and physical interpretations of mathematical concepts, theorems, and theories; and criterion of validity (trueness, truth) of metric geometry, physical concepts, and physical theories is practice;

3) The correct geometrical and physical interpretations of mathematical relationships are the operation of establishing the relation between mathematical and physical quantities. Interpretation is operation of restoration of measure (i.e., interpretation is operation of restoration of the unity of qualitative and of quantitative determinacy) of material object. Mathematics without operation of restoration of measure cannot be used in the natural sciences and is useless intellectual game.

4) The correct metric-geometrical and physical interpretations of mathematical relationships show that a set of standard mathematical functions represents error, mistake, and blunder.

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